updated 1/07/08

University of Wisconsin, Department of Sociology

Sociology 376: Mathematical Models of Social Systems

Spring 2008

Prof. James Montgomery
e-mail: jmontgom@ssc.wisc.edu
website: www.ssc.wisc.edu/~jmontgom

2436 Social Science
Office hours: Friday 9:30-11:30 AM or by appointment

Course Objectives. This course provides an introduction to mathematical models of *social process*, focusing especially on Markov chains and dynamical systems models. Students will learn how to analyze these types of models in order to determine their short-run dynamics and long-run equilibria. Students will make extensive use of mathematical software (Matlab) to compute numerical examples and perform simple simulation analyses. Examples will address a wide range of sociological topics including social mobility, demography, network formation, social influence, cultural evolution, social movements, and residential segregation. This course complements Soc 375 (Introduction to Mathematical Sociology) which explores mathematical models of *social structure*, focusing on social network analysis and related methods.

Prerequisites. To enroll, students should have completed either Soc 375 or a course on matrix algebra (Math 340 or equivalent), or else obtain permission of the instructor. [Soc 375 provides a brief review of matrix algebra that is sufficient for the present course. Students who already know matrix algebra do not need to know social network analysis (or any other topics covered in Soc 375) to enroll in the present course.] Some knowledge of calculus would be helpful, but the course is intended to be accessible to students without calculus. [While more advanced courses on dynamical systems would require calculus, we will make use of graphical or computational methods whenever possible.] Students who have taken math courses covering Markov chains and dynamical systems will already know the relevant mathematics, and students with some background in computer programming may have an advantage. For students who already know matrix algebra, there is no sociology prerequisite, so the course is well-suited for students with other (quantitative) majors.

Evaluation. Grades will be based on two exams (each worth 1/3 of the grade) and weekly problem sets (worth the final 1/3). The midterm exam will be held during class on **Thursday, March 13**; the final exam will be held during exam week on **Tuesday, May 13, 2:45-4:45**.

Exams. The format of the exams will be similar to those given in Soc 375 and Econ 451. [Some of the material in the present course was previously contained in those courses. For some sample exam problems on Markov chains, see the final exams for Soc 375 from 2005 and 2006. For some sample exam problems on dynamical systems, see Exam 3 for Econ 451 from 2005, 2006, and Spring 2007. All of these exams (along with solutions) are posted on my website.] Students should bring calculators to the exams. Graphing calculators (which can multiply matrices) are permitted but not necessary.

Problem sets. Problem sets will be assigned approximately once per week. Problem-set questions are usually more complicated than test questions, and often require the use of Matlab software (see below). Problem sets will be graded on a three-point scale, corresponding roughly to full credit (3), a good-faith effort (2), a bad-faith effort (1), and no effort (0).

Software. In this course, we will make extensive use of the mathematical software package Matlab. While students who have taken Soc 375 will already know Matlab, the present course is intended to be self-contained for students who are not familiar with this software. However, students who do yet know Matlab **must be willing to learn**. Knowledge of Matlab will be necessary to follow the lectures (which are often supplemented with Matlab handouts) and to complete many of the problem sets. We will spend one class period in the Social Science Micro Lab learning this software, but some students may need to spend additional time on their own with the Matlab tutorial to become proficient. Students will be able to use this software in the SSML. [Alternatively, this software is available in some DoIT labs on campus, and can also be purchased from DoIT at a special student price. I'm happy to e-mail m-files to students working outside the SSML.]

Readings. Three books have been ordered for this course:

> Elizabeth S. Allman and John A. Rhodes (2004) *Mathematical Models in Biology: An Introduction*. Cambridge.
> Thomas C. Schelling (1978) *Micromotives and Macrobehavior*. Norton.
> Ian Bradley and Ronald L. Meek (1986) *Matrices and Society*. Pelican.

Although the Bradley and Meek book is out of print, photocopies will be available at the University Book Store. Readings from these three books are indicted by a circle (•) on the reading list below. The remaining readings are contained in a photocopied reader available from the Social Science Copy Center (6120 Social Science).

Honors credit. To receive honors credit for the course, students may write their own problem sets (and provide solutions) for four of the topics on the course outline below. You can choose whichever four topics you wish. Your problems may supplement or extend the problems on the class problem set, but you should also feel free to be more creative, exploring other applications of course material. Some of the honors problems sets may be used as class problem sets in future years.

Changes in the schedule. The tentative course outline is given below. As already indicated, the exam dates are fixed. The precise content of each exam will be announced in class before the exam. More generally, announcements of changes in course material and procedures may from time to time be made in class and students will be responsible for the changes whether present or not. [Because this is the first time this course has been taught, I'm not precisely sure how much of the following material we will be able to cover. To keep the course on schedule, some of the applications below may need to be omitted. Triage decisions will be made as we proceed. If I decide to add any additional readings, they will be provided in class.]

Course outline.

0. Introduction to Matlab

Some instruction will be given in class. However, students not already familiar with Matlab are also strongly encouraged to work through the tutorial contained in Matlab documentation. Begin with the section "Getting Started."

1. Markov chains

Ergodic chains

•Bradley and Meek, Ch 6 ("The Simple Mathematics of Markov Chains") and Ch 7 ("Models of Mobility")

John G Kemeny, J L Snell, and G L Thompson (1966), Ch 4.13 ("Markov Chains"), pp 194-201, and Ch 5.7 ("Application of Matrix Theory to Markov Chains), pp 271-281, in *Introduction to Finite Mathematics*, Prentice-Hall.

•Allman and Rhodes, Ch 2.1 ("Linear Models and Matrix Algebra") and Chs 2.3–2.4 ("Eigenvectors and Eigenvalues")

Applications

Samuel Preston and Cameron Campbell (1993) "Differential Fertility and the Distribution of Traits: The Case of IQ," *American Journal of Sociology* 98(5): 997-1043 [includes comment by Coleman].

James Montgomery (1996) “Dynamics of the Religious Economy,” *Rationality and Society* 8:81-110.

Absorbing chains

•Bradley and Meek, Ch 8 (“The Mathematics of Absorbing Markov Chains”) and Ch 9 (“Everywhere Man is in Chains”)

John G Kemeny, J L Snell, and G L Thompson (1966), Ch 5.8 (“Absorbing Markov Chains”), pp 282-291, in *Introduction to Finite Mathematics*, Prentice-Hall.

Demography

•Bradley and Meek, Ch 10 (“The Seven Ages of Man and Population Problems”)

•Allman and Rhodes, Ch 2.2 (“Projection Matrices for Structured Models”)

Evolution of conventions

H Peyton Young (1996) “The Economics of Convention,” *Journal of Economic Perspectives* 10:105-22.

Herbert Gintis (2000), Ch 10 (“Markov Economies and Stochastic Dynamical Systems,”), pp 220-236, in *Game Theory Evolving*, Princeton.

Network formation

Thomas Fararo and John Skvoretz (1986) “E-State Structuralism: A Theoretical Method,” *American Sociological Review* 51:591-602.

Matthew Jackson and A Watts (2002) “The Evolution of Social and Economic Networks,” *Journal of Economic Theory* 106:265-95.

Network Influence

Noah Friedkin and Eugene Johnsen (1997) “Social Positions in Influence Networks,” *Social Networks* 19:209-222

John G Kemeny, J L Snell, and G L Thompson (1966), Ch 7.2 (“Equivalence Classes in Communication Networks”), pp 394-406, in *Introduction to Finite Mathematics*, Prentice-Hall.

Residential segregation

•Schelling, Ch 4 (“Sorting and Mixing: Race and Sex”), esp pp 147-155.

2. One-dimensional dynamical systems

•Allman and Rhodes, Ch 1 (“Dynamic Modeling with Difference Equations”)

Cultural evolution

Alberto Bisin and Thierry Verdier (2000) “Beyond the Melting Pot: Cultural Transmission, Marriage, and the Evolution of Ethnic and Religious Traits,” *Quarterly Journal of Economics* 115: 955-88.

Critical mass

•Schelling, Ch 3 (“Thermostats, Lemons, and Other Families of Models”)

Mark Granovetter (1978) “Threshold Models of Collective Behavior,” *American Journal of Sociology* 83:1420-1443.

James Montgomery (1989) “Is Underclass Behavior Contagious?” unpublished manuscript.

Street gangs

J Crane, N Boccara, K Higdon (2000) “The Dynamics of Street Gang Growth and Policy Response,” *Journal of Policy Modeling* 22(1):1-25.

Evolutionary game theory

•Schelling, Ch 7 (“Hockey Helmets, Daylight Saving, and Other Binary Choices”)

Larry Samuelson (2002) “Evolution and Game Theory,” *Journal of Economic Perspectives* 16:47-66.

3. Two-dimensional dynamical systems

•Allman and Rhodes, Ch 3 (“Nonlinear Models of Interactions”)

Residential segregation

•Schelling, Ch 4 (“Sorting and Mixing: Race and Sex”), esp pp 155-166.

Mark Granovetter and Roland Soong (1988) “Threshold Models of Diversity: Chinese Restaurants, Residential Segregation, and the Spiral of Silence,” *Sociological Methodology* 18:69-104.

Dyadic interaction

Steven Strogatz (1994), Ch 5.3 (“Love Affairs”), pp 138-140, in *Nonlinear Dynamics and Chaos*, Perseus.

D H Felmlee and D F Greenberg (1999) “A Dynamic Systems Model of Dyadic Interaction,” *Journal of Mathematical Sociology* 23:155-180.

Group interaction

Thomas Fararo (1989) Chs 2.9-2.10 (Simon-Homans model), pp 120-139, in *The Meaning of General Theoretical Sociology*, Cambridge.

Epidemiology

•Allman and Rhodes, Ch 7 (“Infectious Disease Modeling”)